# Intensity dependence of the inverse bremsstrahlung absorption coefficient in hot plasmas



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# Intensity dependence of the inverse bremsstrahlung absorption coefficient in hot plasmas

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**Abstract.** The net absorption coefficient allowing for stimulated emission is derived for intense light in a hot non-relativistic plasma, using a semi-classical approach, which takes into account the non-Maxwellian velocity distribution caused by the strong electric field of the radiation but considers only absorption and emission processes involving a single incident photon. The absorption coefficient is found to vary inversely as both the electric field and the frequency, a result intermediate between the usual weak-field coefficient and a strong-field coefficient due to Rand which is intended to include multi-photon processes. It appears that the net effect of multi-photon processes is to produce stimulated emission. Rand's result is shown to imply that, in order to heat plasma to a given temperature, a minimum time and (on a simple dynamic model) a minimum quantity of plasma are necessary.

## 1. Introduction

Lasers now available are capable of generating pulses of light with peak powers of many gigawatts. By focusing the light, radiation fluxes in excess of $10^{22}$ erg cm$^{-2}$ s$^{-1}$ can be produced. Though much interest has been aroused in the use of laser light to produce very hot dense plasmas, little attention has been given to the determination of the absorption coefficient of a plasma for such intense radiation.

It is generally accepted that, once a moderate degree of ionization has been achieved, the dominant mechanism by which laser light heats a plasma involves free–free transitions of electrons in the fields of positive ions. Many authors (e.g. Basov and Krokhin 1964, Dawson 1964) have assumed that the free–free, or inverse bremsstrahlung, absorption coefficient for light of a given frequency is a function only of the plasma temperature and density, being independent of the radiation flux. However, when the kinetic energy imparted to a plasma electron by the oscillating electric field of the radiation is comparable with, or greater than, the mean thermal energy the electron velocity distribution is far from Maxwellian. Rand (1964) has pointed out that this must have a significant effect on the interaction and has derived an expression for the absorption coefficient in the strong-field case, which varies inversely as the cube of the electric field. As will be shown below, this implies that a certain minimum time is necessary to heat a plasma to a given temperature using light of a given frequency, no matter how much laser power is available. An approximate treatment of the dynamics of small freely expanding spherical plasmas heated in this way shows that Rand's result also implies the need for a certain minimum quantity of plasma if a given temperature is to be attained.

A simple alternative method is used in this paper to calculate the strong-field absorption coefficient for a hot non-relativistic plasma in the low-frequency limit. The electron motion is treated classically and only processes involving a single incident photon are considered. The calculation yields an absorption coefficient which varies inversely as the electric field, a result intermediate between the weak-field expression and Rand's strong-field result. No minimum time or minimum quantity of plasma are required to reach a given temperature. The absorption coefficient is inversely proportional to the frequency of the light, whereas Rand predicts direct proportionality.

## 2. The net free–free absorption coefficient

### 2.1. *General considerations*

Consider a group of $dn_e$ electrons of velocity $v$ and mass $m_e$ interacting in unit volume with $n_i$ stationary heavy ions of charge $Ze$. The rate of spontaneous energy emission at frequency $\nu$ in the interval d$\nu$ by this group is

$$\mathrm{d}S = \mathrm{d}n_e n_i v \sigma(\nu, v) h\nu \, \mathrm{d}\nu \text{ erg cm}^{-3}\text{ s}^{-1}. \tag{1}$$

Here $h$ is Planck's constant and $\sigma(\nu, v)$ is the total emission cross section, summed over polarization directions and integrated over all angles between the initial and final velocities, and between the initial velocity and the direction of the emitted photon. $\sigma(\nu, v)$ may be written (Bethe and Salpeter 1957, p. 419)

$$\sigma(\nu, v) = \frac{32\pi^2 Z^2 e^6}{3^{3/2} m_e{}^2 v^2 c^3 h\nu} G(\nu, v)$$

where $G(\nu, v)$ is the Gaunt factor (see e.g. Bekefi 1966, p. 89) and $c$ the velocity of light.

From the Einstein relations (see e.g. Bekefi 1966, p. 47), the rate of stimulated (or induced) emission in the presence of radiation of energy density $w$ at frequency $\nu$ is given by

$$\mathrm{d}I = \frac{\mathrm{d}n_e n_i c^3}{8\pi h\nu^3} v\sigma(\nu, v) h\nu w \text{ erg cm}^{-3}\text{ s}^{-1}. \tag{2}$$

In the process of emission an electron loses energy $h\nu$ and its velocity changes from $v$ to $v'' = (v^2 - 2h\nu/m_e)^{1/2}$. The *total* rate of absorption of radiant energy by the process in which an electron of velocity $v$ gains energy $h\nu$ may also be obtained from the Einstein relations, and is given by

$$\mathrm{d}A = \frac{\mathrm{d}n_e n_i c^3}{8\pi h\nu^3} \left(\frac{v'}{v}\right)^2 v'\sigma(\nu, v') h\nu w \text{ erg cm}^{-3}\text{ s}^{-1}. \tag{3}$$

Here $v' = (v^2 + 2h\nu/m_e)^{1/2}$, and spontaneous emission has been neglected. Thus the *net* rate of energy absorption by this group of electrons of velocity $v$ is given by

$$\begin{aligned} \mathrm{d}Q &= \mathrm{d}A - \mathrm{d}I \\ &= \frac{4\pi \, \mathrm{d}n_e n_i Z^2 e^6}{3^{3/2} m_e{}^2 h\nu^3 v^2} \{v'G(\nu, v') - vG(\nu, v)\} w \text{ erg cm}^{-3}\text{ s}^{-1}. \end{aligned} \tag{4}$$

## 2.2. *Thermal velocity distribution (weak field)*

For a Maxwellian distribution of electron velocities, integration of expression (4) presents some problems. In the low-frequency high-temperature limit ($h\nu \ll kT$) it is convenient instead first to obtain the total absorption rate $A_T$ from (3) by setting $v' = v$, replacing the Gaunt factor $G$ by a constant average value $\bar{G}(\nu, T)$ and integrating over all velocities. The result, for a total electron density $n_e$ cm$^{-3}$, is

$$A_T = \frac{4 n_e n_i Z^2 e^6}{3 m_e{}^2 h\nu^3} \left(\frac{2 m_e \pi}{3kT}\right)^{1/2} \bar{G}(\nu, T) w \tag{5}$$

since the average value of $1/v$ at temperature $T$ is $(2m_e/\pi kT)^{1/2}$, where $k$ is Boltzmann's constant.

To obtain the net rate of energy absorption per unit volume $Q_T$ it is then necessary to allow for stimulated emission. Thermodynamic considerations (see e.g. Bekefi 1966, p. 53) lead to the expression

$$Q_T = A_T \left\{1 - \exp\left(-\frac{h\nu}{kT}\right)\right\}$$

so that when $h\nu \ll kT$

$$Q_T \simeq \frac{A_T h\nu}{kT}.$$

The time-averaged rate is given by

$$\langle Q_{\mathrm{T}} \rangle = \frac{8}{3}\left(\frac{\pi}{6}\right)^{1/2} \frac{n_e n_i Z^2 e^6}{\nu^2 (m_e k T)^{3/2}} \bar{G}(\nu, T) \langle w \rangle \text{ erg cm}^{-3}\,\text{s}^{-1} \tag{6}$$

where $\langle w \rangle$ is the time average of $w$. The corresponding net absorption coefficient

$$\alpha_{\mathrm{T}} = \frac{\langle Q_{\mathrm{T}} \rangle}{\langle w \rangle c} = \frac{8}{3}\left(\frac{\pi}{6}\right)^{1/2} \frac{n_e n_i Z^2 e^6}{c\nu^2 (m_e k T)^{3/2}} \bar{G}(\nu, T) \text{ cm}^{-1}. \tag{7}$$

For

$$T \gtrsim 10^6\,{}^{\circ}\mathrm{K}, \qquad \bar{G}(\nu, T) \simeq \frac{3^{1/2}}{\pi} \ln\left(\frac{4kT}{\gamma h\nu}\right)$$

(Oster 1961), where $\ln \gamma$ = Euler's constant = 0·577.

2.3. *Non-thermal velocity distribution* (*strong field*)

If the electric field $E$ e.s.u. due to the radiation flux is sufficiently strong for the electron velocities to depart significantly from a Maxwellian distribution, i.e. if

$$(eE)^2 \gtrsim 12\pi^2 m_e k T \nu^2$$

strictly speaking no temperature can be defined when the field is applied. Also the electron velocity and the instantaneous local energy density of the radiation will be to some extent correlated since both are functions of the electric field.

Let us consider a volume element whose dimensions are small compared with the wavelength of the radiation. The local electric field may be written as

$$E = E_0 \cos(2\pi\nu t). \tag{8}$$

At optical frequencies the motion of an electron due to the radiation will have an amplitude small compared with a wavelength even at extremely high power densities. Also, the period of oscillation of the radiation will be much shorter than either its coherence time or the collision time of the electrons. Thus, to a good approximation, the oscillatory component of the electron velocity has the instantaneous value

$$u = u_0 \sin(2\pi\nu t) = \frac{eE_0}{2\pi\nu m_e} \sin(2\pi\nu t)$$

in the direction of the field. This value is common to all electrons in the volume element considered. Additionally, the electrons always have some random energy, which in the absence of the field would correspond to a temperature $T$. For convenience, the random part of the velocity distribution will be replaced by an average velocity $v_{\mathrm{T}}$ which will be supposed to be perpendicular to the electric field (Rand 1964). Then

$$v^2 = u^2 + v_{\mathrm{T}}^2. \tag{9}$$

A hot plasma is considered, such that $v_{\mathrm{T}}^2 \gg 2h\nu/m_e$, so the Born approximation is valid and

$$\left.\begin{aligned} G(\nu, v) &= \frac{3^{1/2}}{\pi} \ln\left(\frac{v+v''}{v-v''}\right) \\ G(\nu, v') &= \frac{3^{1/2}}{\pi} \ln\left(\frac{v'+v}{v'-v}\right) \end{aligned}\right\}. \tag{10}$$

Thus

$$\begin{aligned} v'G(\nu, v') - vG(\nu, v) &= (v'-v)G(\nu, v') + v\{G(\nu, v') - G(\nu, v)\} \\ &\simeq \frac{3^{1/2}h\nu}{\pi m_e v} \ln\left(\frac{2m_e v^2}{h\nu}\right) \end{aligned}$$

since $\ln(2m_e v^2/h\nu) \gg 1$. Hence from (4) the net local rate of absorption of energy per unit volume by $n_e$ electrons, all of velocity $v$,

$$Q = \frac{4n_e n_i Z^2 e^6}{3m_e^3 \nu^2 v^3} \ln\left(\frac{2m_e v^2}{h\nu}\right) w. \tag{11}$$

When $E_0 \simeq 0$, $v \simeq v_T$ and the time average of (11) may be compared with (6). Despite the approximation made in (9), the results agree to within a factor close to $\frac{1}{2}\pi$ provided $v_T$ is taken to equal $(\pi kT/2m_e)^{1/2}$, by analogy with the previous section, rather than the r.m.s. value $(3kT/m_e)^{1/2}$.

The instantaneous value of the radiation energy density in the small volume element considered is given by

$$w = \frac{E_0^2}{4\pi} \cos^2(2\pi\nu t).$$

Thus the instantaneous local value of $Q$ becomes

$$Q_E = F\left[\frac{E_0^2 \cos^2(2\pi\nu t)}{[\{u_0 \sin(2\pi\nu t)\}^2 + v_T^2]^{3/2}}\right] \ln\left[\frac{2m_e}{h\nu}[\{u_0 \sin(2\pi\nu t)\}^2 + v_T^2]\right] \tag{12}$$

where

$$F = \frac{n_e n_i Z^2 e^6}{3\pi m_e^3 \nu^2}.$$

The time average of $Q$ is then given by

$$\langle Q_E \rangle = \frac{2}{\pi} F \frac{E_0^2}{u_0^3} \int_0^{\pi/2} \frac{\cos^2\theta}{(\sin^2\theta + a^2)^{3/2}} \ln\left\{\frac{2m_e u_0^2}{h\nu}(\sin^2\theta + a^2)\right\} d\theta \tag{13}$$

where $a = v_T/u_0$ and $\theta = 2\pi\nu t$. In sufficiently strong fields, such that $a \ll 1$, the main contribution to the integral comes from small values of $\theta$. (This consideration justifies the use of the total emission cross section instead of the differential cross section for particular orientations.) Thus when $\frac{1}{2}mu_0^2 \gg \frac{1}{2}mv_T^2 \gg h\nu$

$$\begin{aligned} \langle Q_E \rangle &\simeq \frac{2}{\pi} F \frac{E_0^2}{u_0^3} \ln\left(\frac{2mv_T^2}{h\nu}\right) \int_0^{\pi/2} \frac{\cos^2\theta \, d\theta}{(\sin^2\theta + a^2)^{3/2}} \\ &= \frac{16}{3\pi}\left(\frac{2}{\pi}\right)^{1/2} \frac{n_e n_i Z^2 e^5 \langle w \rangle^{1/2}}{\nu mkT} \ln\left(\frac{\pi kT}{h\nu}\right) \end{aligned} \tag{14}$$

where $\langle w \rangle = E_0^2/8\pi$ is the time-averaged radiant energy density. The corresponding net absorption coefficient

$$\alpha_E = \frac{16}{3\pi}\left(\frac{2}{\pi}\right)^{1/2} \frac{n_e n_i Z^2 e^5}{c\nu m_e kT \langle w \rangle^{1/2}} \ln\left(\frac{\pi kT}{h\nu}\right) \text{cm}^{-1}. \tag{15}$$

Rand (1964) employed a rather more elaborate procedure intended to include multi-photon processes. If we replace Rand's $v_0$ by $(\pi kT/2m_e)^{1/2}$, his result for strong fields and low temperatures ($\frac{1}{2}mu_0^2 \gg h\nu \gg \frac{1}{2}mv_0^2$) may be written

$$\alpha_{RL} = \left(\frac{\pi}{2}\right)^{1/2} \frac{n_e n_i Z^2 e^3 \nu}{c\langle w \rangle^{3/2}} \ln\left(\frac{4e^2\langle w \rangle}{\pi^2 m_e kT \nu^2}\right) \ln\left\{\frac{32\pi e(kT)^{3/2}\langle w \rangle^{1/2}}{m_e^{1/2} h^2 \nu^3}\right\} \text{cm}^{-1}.$$

For strong fields and high temperatures ($\frac{1}{2}mu_0^2 \gg \frac{1}{2}mv_0^2 \gg h\nu$) Rand's treatment must be modified by changing the lower limit of integration over the longitudinal wave vector, $k_{\min}$ in his notation, which becomes approximately $2\pi mv_0/h$. Then $\ln(k_0/k_{\min}) \simeq 1$ at the time when the absorption rate is greatest. Thus, for the conditions under which expression (15) is valid, Rand's procedure gives a net absorption coefficient

$$\alpha_{\mathrm{RH}} = 2(2\pi)^{1/2}\frac{n_e n_i Z^2 e^3 \nu}{c\langle w\rangle^{3/2}}\ln\left(\frac{64e^2\langle w\rangle}{\pi^2 m_e k T\nu^2}\right)\mathrm{cm}^{-1}. \tag{16}$$

## 3. Implications for plasma heating

Let us consider a parallel laser beam of radiant flux $D$ erg cm$^{-2}$ s$^{-1}$. The power absorbed per unit volume in a thin sheet of plasma perpendicular to the direction of propagation of the beam is $\alpha D$ erg cm$^{-3}$ s$^{-1}$.

For a small radiant flux in a high-temperature plasma, i.e. for

$$D \ll \frac{3\pi c m_e k T\nu^2}{2e^2}, \qquad kT \gg h\nu$$

expression (7) may be used to find the power absorbed per unit volume

$$\alpha_{\mathrm{T}} D = \frac{4}{3}\left(\frac{2}{\pi}\right)^{1/2}\frac{n_e n_i Z^2 e^6}{c(mk)^{3/2}}\ln\left(\frac{4kT}{\gamma h\nu}\right)\frac{D}{\nu^2 T^{3/2}}. \tag{17}$$

For a large radiant flux in a hot plasma, i.e.

$$D \gg \frac{3\pi c m_e k T\nu^2}{2e^2}, \qquad kT \gg h\nu$$

expression (15) derived in this paper for the absorption coefficient gives

$$\alpha_{\mathrm{E}} D = \frac{16}{3\pi}\left(\frac{2}{\pi}\right)^{1/2}\frac{n_e n_i Z^2 e^5}{c^{1/2} m_e k}\ln\left(\frac{6kT}{h\nu}\right)\frac{D^{1/2}}{\nu T} \tag{18}$$

while expression (16) derived from Rand's paper gives

$$\alpha_{\mathrm{RH}} D = 2(2\pi)^{1/2} c^{1/2} n_e n_i Z^2 e^3 \ln\left(\frac{64e^2 D}{\pi^2 m_e k T c\nu^2}\right)\frac{\nu}{D^{1/2}}. \tag{19}$$

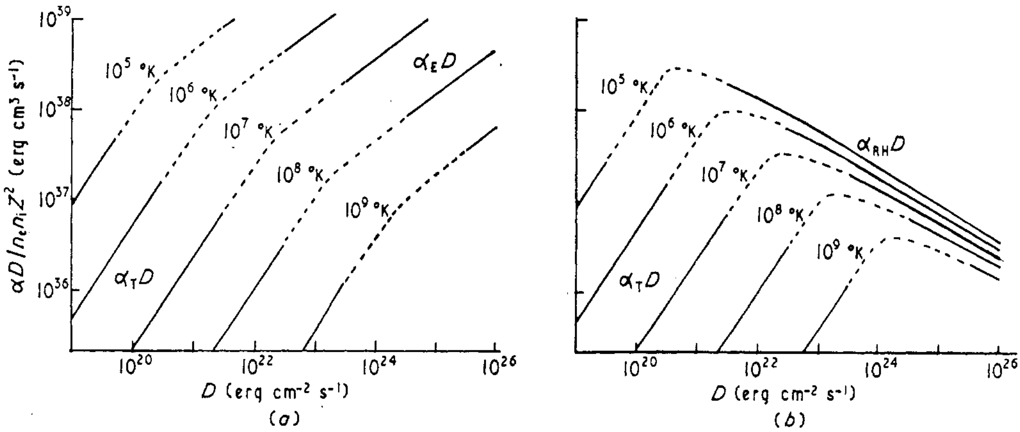


Figure 1. Power absorbed per unit volume of plasma (normalized to unit ion and electron densities, for ion charge $Z = 1$) from a neodymium laser beam of radiant flux $D$: (*a*) as calculated in this paper; (*b*) as derived from Rand (1964).

In figures 1(*a*) and 1(*b*), $\alpha_{\mathrm{E}} D/n_e n_i Z^2$ and $\alpha_{\mathrm{RH}} D/n_e n_i Z^2$ are plotted as functions of $D$ at several temperatures for the frequency of the neodymium laser ($\nu = 2{\cdot}83\times10^{14}$ Hz). In each case $\alpha_{\mathrm{T}} D/n_e n_i Z^2$ is also plotted in the region of small values of $D$. Broken lines show the estimated behaviour of the curves in the transitional regions.

As may be seen from figure 1(*a*), if the strong-field absorption coefficient is correctly given by $\alpha_{\mathrm{E}}$, the power absorbed by the plasma sheet increases monotonically with $D$. If, on the other hand, the correct strong-field absorption coefficient is $\alpha_{\mathrm{RH}}$, it is clear from figure 1(*b*) that the power absorbed by a plasma at a given temperature will have a maximum value, reached when $u_0 \simeq v_{\mathrm{T}}$. There will then be an optimum power density for plasma heating, given by

$$D_{\mathrm{opt}} \simeq \frac{\pi^2 c m_{\mathrm{e}} k T \nu^2}{4e^2}. \tag{20}$$

For the neodymium laser $D_{\mathrm{opt}} \simeq 3\times 10^{15} T \,\mathrm{erg\,cm^{-2}\,s^{-1}}$ at $T$ °K.

The maximum power which can be absorbed per unit volume of plasma will be rather less than $\alpha_{\mathrm{T}} D_{\mathrm{opt}}$ (and will be independent of the laser frequency except through slowly varying logarithmic terms). It is shown in the appendix that there is a minimum time needed to produce plasma at a given temperature by this process if Rand's treatment is correct, no matter how much laser power and energy are available. It is also shown that on a simple dynamic model a minimum quantity of plasma is necessary. Thus the absorption coefficient $\alpha_{\mathrm{RH}}$ derived following Rand imposes more severe restrictions on the experimental conditions necessary for very high temperatures to be achieved than are imposed by $\alpha_{\mathrm{E}}$.

## 4. Discussion

The net absorption coefficient $\alpha_{\mathrm{E}}$ derived in § 2.3 only takes into account processes involving a single incident photon, whereas Rand's $\alpha_{\mathrm{RL}}$ and its high-temperature equivalent appear to include the effects of multi-photon processes. If this is the reason for the difference between the results, it seems that multi-photon processes reduce the net absorption coefficient for strong radiation fields, because $\alpha_{\mathrm{RH}} < \alpha_{\mathrm{E}}$. Thus, if we take all multi-photon processes together, the rate of stimulated emission apparently exceeds the total rate of absorption.

For certain directions of the electron velocity relative to the electric field of the radiation Marcuse (1962, 1963) predicted that stimulated emission will exceed absorption in weak fields. Bunkin and Fedorov (1966) considered multi-photon effects in the weak- and strong-field limits for electron velocities both parallel and perpendicular to the electric field. They confirmed Marcuse's result for weak fields, but for strong fields were unable to determine whether or not emission exceeds absorption. It is clear from the results obtained above that a knowledge of the correct net absorption coefficient is of some practical importance, and a more general treatment of the problem is desirable.

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## Appendix. Dynamics of laser-heated plasma

Some of the consequences of using the absorption coefficient derived from Rand's paper are discussed below for the case of an isolated speck of matter in vacuum, heated rapidly by a laser pulse to high temperatures.

The model adopted is similar to that of Basov and Krokhin (1964), Dawson (1964) and others. Spherical symmetry is assumed. The early stages of plasma formation are ignored, and at time $t = 0$ the target is supposed to be a cold uniform fully ionized sphere of radius $r_0$. The initial electron density $n_{\mathrm{e}_0}$ is determined by the condition that the plasma frequency equals the laser frequency, i.e.

$$n_{\mathrm{e}_0} = \frac{\pi m_{\mathrm{e}} \nu^2}{e^2}. \tag{A1}$$

The plasma expands rapidly to lower densities, however, so no correction involving the plasma frequency is made to the absorption coefficient. Radiation losses are neglected.

When the plasma is heated and begins to expand, an increasing part of the energy supplied to it takes the form of radially directed kinetic energy. The dynamics of this process are simplified by the assumptions that the plasma is isothermal and of uniform density. The equations of conservation of energy and momentum are then

$$\left.\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}(M\dot{r}^2+3NkT) &= 2P \qquad \text{(A2)} \\ M\ddot{r}-4\pi r^2 p &= 0 \qquad \text{(A3)} \end{aligned}\right\}$$

where $N$ is the total number of particles ($= N_e+N_i$), $M$ = three-fifths of total mass of plasma (Dawson 1964), $p = nkT$, where $n = n_e+n_i$, and $P$ is the rate at which energy is supplied to the plasma. Basov and Krokhin took $P$ to be a constant. In the following analysis, in order to find a lower limit for the time needed to heat the plasma to a given temperature, it will be supposed that the maximum rate of energy absorption per unit volume derived in § 3 on the basis of Rand's treatment is maintained over the whole plasma at all times. Thus

$$P = B\frac{N^2}{r^3T^{1/2}} \qquad \text{(A4)}$$

where

$$B = \left(\frac{\pi}{8}\right)^{1/2}\frac{Z^3e^4}{(m_ek)^{1/2}(Z+1)^2}\ln\left(\frac{4kT}{\gamma h\nu}\right). \qquad \text{(A5)}$$

(A single ion species of charge $Ze$ is assumed.) The slowly varying logarithmic term is replaced by a constant which may be taken to be about 8 for the case of neodymium laser light in the range of temperature of interest ($10^6 \lesssim T \lesssim 10^8$ °K). If we take $B$ to be a constant, equations (A2) and (A3) with (A4) give

$$BNt = kr^3T^{3/2} \qquad \text{(A6)}$$

and a solution to these equations is

$$\left.\begin{aligned} r^3 &= \frac{BN}{k}\left(\frac{g+t^2}{f}\right)^{3/2} \qquad \text{(A7)} \\ T &= \frac{t^{2/3}f}{g+t^2} \qquad \text{(A8)} \end{aligned}\right\}$$

where

$$f = r_0\left(\frac{BN}{k}\right)^{1/6}\left(\frac{2M}{3^{1/2}\pi Nk}\right)^{3/4}$$

and

$$g = r_0^3\left(\frac{k}{BN}\right)^{1/2}\left(\frac{2M}{3^{1/2}\pi Nk}\right)^{3/4}.$$

For given $r_0$, $M$ and $N$, the temperature $T$ passes through a maximum $T_{max}$, which occurs at time $t_{max}(T)$ such that $2\{t_{max}(T)\}^2 = g$. It is found that

$$\begin{aligned} T_{max} &= \frac{2^{2/3}}{3r_0}\left(\frac{BN}{k}\right)^{1/2}\left(\frac{2M}{3^{1/2}\pi Nk}\right)^{1/4} \\ &\simeq \frac{4\pi}{3}\left\{\frac{3^{1/2}m_i(Z+1)}{5\pi kZ^2}\right\}^{1/4}\left(\frac{m_eB}{3e^2k}\right)^{1/2}\nu r_0^{1/2} \qquad \text{(A9)} \end{aligned}$$

since $M/N \simeq 3m_i/5(Z+1)$ because $m_i \gg m_e$, and

$$N = \tfrac{4}{3}\pi^2 r_0{}^3 \left(1+\frac{1}{Z}\right)\frac{m_e \nu^2}{e^2}$$

from (A1).

For a given laser frequency, therefore, the maximum temperature attained depends on the initial radius of the plasma. Conversely, a minimum initial radius $(r_0)_{\min}$ is necessary for a specified temperature to be reached, no matter how much laser power and energy are available. From (A9), if a deuterium plasma is heated by a neodymium laser,

$$(r_0)_{\min} = 9 \times 10^{-18} T^2 \qquad \text{(A10)}$$

so to reach a temperature of $4 \times 10^8$ °K the minimum mass of deuterium required is 0·04 g.

The time $t_{\max}(T)$ required to reach the maximum temperature for a given $r_0$ is longer than the time required to reach the same temperature starting with any larger initial radius. There is indeed a minimum time $t_{\min}(T)$ required to reach a given temperature $T$, however large the initial radius and however great the power and energy available. It is given by

$$t_{\min}(T) = \left(\frac{gT}{f}\right)^{3/2}. \qquad \text{(A11)}$$

Expression (A8) is plotted in figure 2 for several values of $r_0$, together with the asymptotic expression (A11), for a deuterium plasma heated by a neodymium glass laser.

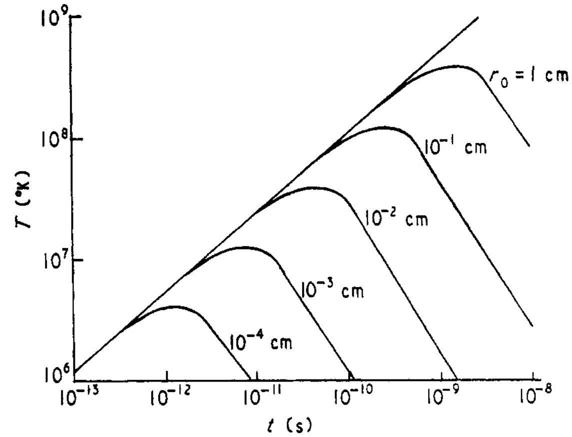


Figure 2. Temperature of deuterium plasma heated by a neodymium laser of optimum radiant flux, plotted as function of time for several initial radii $r_0$.

It must be stressed that expressions (A10) and (A11) are extreme minimum values and would probably need to be increased by a factor of at least 10 in practice.

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